# Bioinformatik für Biologen 

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## Genome Sequencing, Assembly and NGS Data

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Aim: reconstruct template sequence from reads

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## Reference-based Genome Assembly

- If a reference genome of the sequenced species exists (or a relatively close taxonomic relative), we can use it to guide the assembly.
- The reads are mapped to the reference genome using approximative search algorithms.
- The closer the reference is to the sequenced genome, the easier is the mapping and assembly.
- From mapped contiguous reads we construct consensus sequences the contigs.


## Why re-sequencing

Why re-sequencing if a close reference genome already exists?

- Typically one does not re-sequence exactly the same individual the reference originated from.
- Usually one uses the reference to find
- the differences in an individual carrying a disease (e.g. personalized medicine),
- the characteristic changes in a new infectious virus (epidemiology),
- the abundance of alleles in a population (population genetics),
- or just to make the assembly of the (yet unassembled) genome of a related species a little bit easier.


## de novo Genome Assembly

- If no reference genome exists, assembling the sequenced genome is much harder.
- We have to find overlapping reads to stitch them together to longer and longer contigs.


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- discard the trivial matches (i.e. read $i$ matches itself)
- each pair only once (result of $i$ vs $j$ should be identical to $j$ vs $i$ )


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## super-contig / scaffold

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(15) fill the gaps with N's according to the insert sizes used when preparing the sequencing library
(10) the joined contigs are called super-contigs or scaffolds

## Assembly Completeness and Contig Location

- usually it is not possible to easily assemble each chromosome into a single contig or scaffold
(e.g. due to repeats, low quality regions, too low read coverage)
- thus, it can be important to locate scaffolds in the genome using, e.g., FISH (fluorescence in-situ hybridization) with genetic markers.

Source: Westbrook et al. (2008)
red chromosome marker, green probe


## Assembly from 2nd and 3rd generation sequencing reads

- CAP3 has been developed for Sanger sequencing reads.
- NGS reads are typically shorter and come in huge numbers.
- Thus, also the overlaps are short, producing false positives easily.
- Assembly of NGS data works along the same principles.
- However they have to employ more elaborate methods to deal with the amount of data, the short overlaps and to efficiently detect false positive overlaps.
- A number of such tools apply approaches like de Bruijn graphs.


## Excursion: Beginning of Graph Theory




Leonhard Euler (1707-1783)
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## Euler Tour or Circuit or Cycle

is a path visiting each edge (of a graph) exactly once and ending at the starting point.

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123 SOLVTIO PROBLEMATIS
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gEOMETRIAM SITVS
PERTINENTIS.
AVCTORE
Leonh. Eilero.

## §. I .

Tibula viII,Raeter illam Geometriae partem, quac circa guptitates verfatur, et omni tempore fummo fipdio eft excala, alterius partis etiamnum admodum ignotae primus mentionem fecit Leibnitsius, quam Geometriam fitus vocanit. Ita pars ab ipto in folo gytu determinando, fitusque proprieratibus ervendis occupata effe ftatuitur; in quo negotio neque ad quantitgtes re-〔piciendum, veque calcalo quantitatum vrendum fit. Criusmodi autern problemata ad lianc fitus Gcometriam pertineant, et quali methodo in is refoluendis vti oporteat, noo fatis eft definitum. Quamobrem, cum naper problematis cuiasdam mentio effet facta, quod quidem ad geornetriam pertincre videbatur, at ita erat comparatum, vt neque determinationem quantitatum requirerer, neque folutionem calculi quantitanm ope admitteret, id $\cdot$ ad geomerriam fions referre hand dubitani pracfertim crod in eius folutione folns fitus in confide praciertion chad alobs vero nollius prorfus conide rationem venat, calculus vero nollius prorfus
Methodum ergo meam quam ad huius generis proble-

## Euler Paths and Tours on directed graphs

In directed graphs, that means, that edges have only one direction in which they can be crossed. . .

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## Euler Path

exists iff there at most one node with in-degree - out-degree $=1$ exists at most one node with out-degree - in-degree $=1$.

Note, in graph theory the terms node and vertex (pl. vertices) are used interchangeably, as are directed edge and arc.

## Variation: Hamiltonian Paths and Tours



William Rowan Hamilton
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- Hamiltonian Tours are a special case of the Traveling Salesman Problem.


## Nicolaas de Bruijn

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- Find the shortest circular superstring that contains all possible $k$-mers as substrings.


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- For example, the number DNA-triplets:
$n^{k}=4^{3}=64$.


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- edges: directed links between nodes $\mathbf{a}$ and $\mathbf{b}$ if the $k$ - 2-long prefix of $\mathbf{b}$ is the suffix of $\mathbf{a}$.


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- nodes: for all possible $(k-1)$-mers
- edges: directed links between nodes $\mathbf{a}$ and $\mathbf{b}$ if the $k-2$-long prefix of $\mathbf{b}$ is the suffix of $\mathbf{a}$.
- Note: A Eulerian Tour exists because every node have one in-edge and one out-edge for each character in $\Sigma$.


## De Bruijn Graph Example

De Bruijn Graph for $k=4$ and $\Sigma=\{0,1\}$ :


- The nodes are labeled with $000,001,010,011$, 100, 101, 110, 111.
- The edges are labeled linking the overlapping node labels, e.g. $100 \rightarrow$ 001 with 1001.


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- The Eulerian Tour (marked by blue numbers) spells out the circular superstring: 0000110010111101.


## De Bruijn Graphs in Sequence Assembly



## De Bruijn Graphs in Sequence Assembly

a


$V$
ATGGCGT
11111
GGCGTGC
$\begin{array}{llll}1 & 1 & 1 & 1 \\ C G T G C A A\end{array}$
11111
TGCAATG
CAATGGC
111।
ATGGCGT
Genome: ATGGCGTGCAATGGCGT

## De Bruijn Graphs in Sequence Assembly

C


## $\bar{V}$

ATGGCGT
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Genome: ATGGCGTGCAATG

## Sequence De Bruijn Graph (from reads to graph)

read 1:
$\begin{array}{lllllllll}\mathbf{G} & \mathbf{G} & \mathbf{A} & \mathbf{C} & \mathbf{T} & \mathbf{A} & \mathbf{A} & \mathbf{A} & \mathbf{T}\end{array}$

- construct a de Bruijn graph for each read with $k-1=3$


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G G A
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```
\longrightarrow-O-O-O
GGA GAC ACT CTA TAA AAA AAT
(1) (1) (1) (1) (1) (1)
```

```
GAC ACC CCA CAA AAA AAT ATC
```

- take all 'read graphs'


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- ... and merge identical nodes


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- ...gaining a large de Bruijn graph
- sum up the read counts at the merged node
- do repeat this for all other reads


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- delete the obsolete nodes and re-link the branching nodes
- remove the tips and bulges with low read counts (sequencing errors)
- re-compactify the graph
- all the labels on the non-branching paths are our reconstructed contigs


## De Bruijn Graphs for Assembly with Sequencing Errors



- The number of reads participating in bulges and tips tell us which are the frequent, and thus likely true ones.


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- Bulges and tips with few reads are removed.


## De Bruijn Graphs for Assembly with Repeats

## AAGACTCCGACTGGGACTTT



A de Bruijn graph of a sequence


B condensed de Bruijn graph

- In the case of repeats, Euler paths are not possible, because the edges of the repeated region have to be used repeatedly.


## De Bruijn Graphs for Assembly with Repeats

## AAGACTCCGACTGGGACTTT



A de Bruijn graph of a sequence


Source: Chaisson et al. (2009)

- In the case of repeats, Euler paths are not possible, because the edges of the repeated region have to be used repeatedly.
- Typically the order in which edges from a repeat have to be followed cannot be determined. Then the paths have to be kept as separate contigs.


## De Bruijn Graphs for Assembly with Repeats

## AAGACTCCGACTGGGACTTT



A de Bruijn graph of a sequence


Source: Chaisson et al. (2009)

- In the case of repeats, Euler paths are not possible, because the edges of the repeated region have to be used repeatedly.
- Typically the order in which edges from a repeat have to be followed cannot be determined. Then the paths have to be kept as separate contigs.
- Exception: if we have reads or paired-end information, which reach longer than the repeat, this helps to order the contigs.


## Bidirected de Bruijn Graphs (Medvedev et al. 2007)

seq1: GGACTAAAT

- Problem: How do we know which strand our read is from? We don't!


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- With bidirected de Bruijn Graphs one can cover $k$-mers and their complements.


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## Bidirected de Bruijn Graphs (Medvedev et al. 2007)



- Problem: How do we know which strand our read is from? We don't!
- With bidirected de Bruijn Graphs one can cover $k$-mers and their complements.
- Note:

At no time two nodes with identical strings can exist in one (sub)graph, and both strings have to be treated equally.

## Some measures on genome sequencing and assembly

## Coverage

Coverage describes the average number of times a nucleotide in the template DNA has been sequenced which is equivalent to the number of reads that cover each nucleotide on average.

$$
\text { coverage }=\frac{\left.\sum_{i \in\{a l l} \text { reads }\right\}}{\text { length of template or genome }}
$$

Rule of thumb: the higher the better!

## Some measures on genome sequencing and assembly

The quality of an assembly is hard to measure. Typically several values are used like

- maximum contig/scaffold length
- average contig/scaffold length
- combined total length
- the N50 or the NG50 value


## N50 and NG50

## N50 value

All contigs/scaffold are ordered descending in size. Starting from the largest contig/scaffold add their lengths. The N50 value is the length of the first contig, for which this sum of contig lengths covers $\geq 50 \%$ of the total length of contigs/scaffolds, i.e. the entire assembly.

Rule of thumb: the longer the better!

## NG50 value

All contigs/scaffold are ordered descending in size. Starting from the largest contig/scaffold add their lengths. The NG50 value is the length of the first contig, for which this sum of contig lengths covers $\geq 50 \%$ of the total length of the sequenced genome.

Rule of thumb: the longer the better!
Sometimes other percentages than $50 \%$ are used leading to, e.g. N70 etc.

## Reference based assembly

The whole procedure gets much easier if we have a reference genome available.

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## Reference based assembly

The whole procedure gets much easier if we have a reference genome available.

- mapping using search tools like BLAST or BLAT
- dynamic programming (e.g. Smith-Waterman) with pre-filtering to keep the candidate regions small, using
- hash-based k-mer index
- spaced-seeds index
- Approaches using the Burrows-Wheeler-Transform (BWT) of the reference sequence,
and the mapped reads are then summarized to contigs using consensus approaches.


## Hash-based approaches

- Hash-based approaches typically require matching seed sequences (one or several) to identify candidate regions to be checked (similar to Baeza-Yeates-Perleberg)


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- often contiguous seeds are used (e.g. perfectly matching words of length $k$ )
... ACTATCATCGTACACAT... reference sequence
АСтАтСАтTGTACACAT query sequence


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$$
111111111 \quad \text { seed encoding }
$$

... ACTATCATCGTACACAT... reference sequence
TCATCGTAC seed sequence (len=9)
ACTATCATTGTACACAT query sequence

## Hash-based approaches

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- often contiguous seeds are used (e.g. perfectly matching words of length $k$ )

111111111 seed encoding
... AСTATCATCGTACACAT... reference sequence
АСТАТСАТС seed sequence (len=9)
ACTATCATTGTACACAT query sequence

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- often contiguous seeds are used (e.g. perfectly matching words of length $k$ )

111111111 seed encoding
... ACTATCATCGTACACAT... reference sequence
C G T A C A C A T seed sequence (len=9)
ACTATCATTGTACACAT query sequence

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111111111 seed encoding
... ACTATCATCGTACACAT... reference sequence
C GTACACAT seed sequence (len=9)
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- Another way is to use spaced seeds, i.e. that only certain letters in a longer word have to match.


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... ACTATCATCGTACACAT... reference sequence
C GTACACAT seed sequence (len=9)
ACTATCATTGTACACAT query sequence

- Another way is to use spaced seeds, i.e. that only certain letters in a longer word have to match.

10001110001110011 seed encoding
... ACTATCATCGTACACAT... reference sequence
A . . . TC A . . T A C . . A T spaced seed (weight=9, len=17)
ACTATCATTGTACACAT query sequence

## Hash-based approaches

- Hash-based approaches typically require matching seed sequences (one or several) to identify candidate regions to be checked (similar to Baeza-Yeates-Perleberg)
- often contiguous seeds are used (e.g. perfectly matching words of length $k$ )

$$
111111111 \text { seed encoding }
$$

АСТАТСАТСGTACACAT... reference sequence C GTACACAT seed sequence (len=9)
ACTATCATTGTACACAT query sequence

- Another way is to use spaced seeds, i.e. that only certain letters in a longer word have to match.

$$
\begin{array}{rllllllllllllll}
1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & \text { seed encoding } \\
\ldots \text {... } C \text { T ATC }
\end{array}
$$

A . . . TCA... TAC.. A T spaced seed (weight=9, len=17)
ACTATCATTGTACACAT query sequence

- It has been shown that the use of spaced seeds is much more sensitive, missing less hits. Especially, when using sets of spaced seeds.


## Hash-based mapping



- mapping with candidate filtering based on (spaced) seed matches is easy to implement
- however, to generate a typical seed index is memory-intense (about 50GB for the human genome of 3 Gbp )
- the example uses six spaced seeds
(1111111100000000, 00000000111111111, 0000111100001111, 1111000011110000, 0000111111110000, 1111000000001111)
- from all candidates the actual best hit position of the read has to be found by alignment and reported


## Burrows-Wheeler-Transform (BWT) - encoding

Methods using the Burrows-Wheeler-Transform (BWT) for mapping generate the BWT from the text, e.g. the genome, adding an start ( ${ }^{\wedge}$ ) and end character (\$): (Note, in the example we assume that ^ and \$ sort after the letters.)
use text with start+end:
^ BANANA\$

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^ BANANAS
^ BANANA\$

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(Note, in the example we assume that ^ and \$ sort after the letters.)
use text with start+end: generate all rotations:

$$
\begin{aligned}
& \hat{Y} \text { BANANAS } \\
& \text { SABANAN }
\end{aligned}
$$

^ BANANA\$

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(Note, in the example we assume that ^ and \$ sort after the letters.)
use text with start+end: generate all rotations:

$$
\begin{aligned}
& \wedge \text { BANANA } \\
& \$ \wedge B A N A N A \\
& A \$ \wedge B A N A N
\end{aligned}
$$

^ BANANA\$

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(Note, in the example we assume that ^ and \$ sort after the letters.)
use text with start+end:
generate all rotations:
^ B ANANAS
$\${ }^{*} B A N A N A$
A \$ ^BANAN
^BANANA\$ NA\$^BANA

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(Note, in the example we assume that ^ and \$ sort after the letters.)
use text with start+end: generate all rotations:

|  |
| :---: |
|  |
| A \$ ${ }^{\text {a }}$ B |
| A |
| ANA\$ ${ }^{\text {¢ }}$ BAN |
|  |
|  |
|  |

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use text with start+end:

| generate all rotations: ^ B A N A A \$ | sort lexicographically: |
| :---: | :---: |
| \$ ^ BANANA | ANA \$ ${ }^{\wedge} \mathrm{BAN}$ |
| A \$ ${ }^{\text {A B ANAN }}$ | A \$ ${ }^{\text {c }} \mathrm{BANAN}$ |
| NA\$ ^ BANA | BANANA\$ |
| ANA\$ ${ }^{\wedge} \mathrm{BAN}$ | NANA\$ ${ }^{\text {A }}$ BA |
| NANA \$ ^ B A | NA \$ ${ }^{\text {A }}$ BANA |
| ANANA \$ ${ }^{\wedge} \mathrm{B}$ | ${ }^{\wedge} \mathrm{BANANA} A \$$ |
| BANANA\$ | \$ ${ }^{\text {A }} \mathrm{BANANA}$ |

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use text with start+end: generate all rotations:


BWT is the last column:

BNN^AA\$A

## Burrows-Wheeler-Transform (BWT) - encoding

Methods using the Burrows-Wheeler-Transform (BWT) for mapping generate the BWT from the text, e.g. the genome, adding an start ( ${ }^{\wedge}$ ) and end character (\$): (Note, in the example we assume that ^ and \$ sort after the letters.)

| use text with start+end: | generate all rotations: | sort lexicographically: | BWT is the last column: |
| :---: | :---: | :---: | :---: |
|  | $\wedge^{\text {B A N A N A }}$ | ANANA \$ ^ B |  |
|  | \$ ${ }^{\text {a }} \mathrm{BANANA}$ | ANA\$ ${ }^{\text {A }} \mathrm{BAN}$ |  |
|  | A ${ }^{\wedge}{ }^{\wedge} \mathrm{BANANN}$ | A \$ ${ }^{\text {a B A N AN }}$ |  |
| ^BANANA \$ | NA\$^BANA $\text { ANA\$ }{ }^{\wedge} \mathrm{BAN}$ | BANANA\$^ NANA\$ ^ BA | B N N ${ }^{\text {A }}$ A A \$ A |
|  |  | NASヘBANA |  |
|  |  | $\wedge^{\wedge} \mathrm{BANANA}$ ( |  |
|  | BANANA\$ | \$ ^BANANA |  |

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| use text with start+end: | generate all rotations: | sort lexicographically: | BWT is the last column: |
| :---: | :---: | :---: | :---: |
|  | $\wedge^{\text {B A A A N A }}$ | A NANA ${ }_{\text {¢ }}$ ^ B |  |
|  | \$ ${ }^{\text {a }} \mathrm{BANANA}$ | ANA\$^BAN |  |
|  | A ${ }^{\wedge}{ }^{\wedge} \mathrm{BANANN}$ | A\$ ${ }^{\wedge} \mathrm{BANAN}$ |  |
| ^ BANANA\$ | NA\$^BANA $\text { ANA\$ }{ }^{\wedge} \mathrm{BAN}$ | BANANA\$ NANA§^BA | BNN^AA\$A |
|  |  | NASへ BANA |  |
|  |  | ${ }^{\wedge} \mathrm{B} A N A N A \$$ |  |
|  | BANANA\$ | \$ ^ BANANA |  |

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| use text with start+end: | generate all rotations: | sort lexicographically: | BWT is the last column: |
| :---: | :---: | :---: | :---: |
| ${ }^{\wedge} \mathrm{BANANA}$ ( | $\wedge^{\text {B }}$ ANANA \$ | A NANA\$ ^ B |  |
|  | \$ ^ BANANA | ANA\$ ${ }^{\text {A }}$ B AN |  |
|  | A \$ ^ BANAN | A \$ ^ BANAN |  |
|  | NA\$ $\mathrm{A}_{\mathrm{B}}^{\mathrm{B}} \mathrm{ANA}$ ANA\$^BAN | BANANA\$ ${ }^{\wedge}$ NANA§ ${ }^{\wedge} B A$ | BNN^AA\$A |
|  | NANA ${ }^{\text {A }}$ ^ BA | NA\$^BANA |  |
|  |  | ${ }^{\wedge} \mathrm{BANANA}$ ( |  |
|  | BANANA\$ | \$ ${ }^{\text {a }} \mathrm{BANANA}$ |  |

Originally, the Burrows-Wheeler-Transform (BWT) has been introduced in the field of data compression, because (a) the BTW compresses better than the original text and (b) one can decode the original text from the BWT.

## Burrows－Wheeler－Transform（BWT）－decoding

From the BTW the original text can easily decoded：
start from
the BTW
ゆひゆゆンZZ四

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:
start from

| the BTW | sort |
| :---: | :---: |
| $\mathbf{B}$ | $\mathbf{A}$ |
| $\mathbf{N}$ | $\mathbf{A}$ |
| $\mathbf{N}$ | $\mathbf{A}$ |
| $\mathbf{A}$ | $\mathbf{B}$ |
| $\mathbf{A}$ | $\mathbf{N}$ |
| $\mathbf{A}$ | $\mathbf{N}$ |
| $\mathbf{\$}$ | $\mathbf{N}$ |
| $\mathbf{A}$ | $\mathbf{\$}$ |

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:

| start from the BTW | sort | we know 1st +last column |
| :---: | :---: | :---: |
| B | A | A . . . B |
| N | A | A . . . N |
| N | A | A . . . N |
| $\wedge$ | B | B . . . ${ }^{\wedge}$ |
| A | N | N . . . A |
| A | N | N . . . A |
| \$ | ヘ | \$ |
| A | \$ | \$ . . . A |

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:

| start from the BTW | sort | we know 1st +last column |  | rotate BTW front) |
| :---: | :---: | :---: | :---: | :---: |
| B | A | A . . B | B | A |
| N | A | A . . N | N | A |
| N | A | A . . . N | N | A |
| $\wedge$ | B | B ... ${ }^{\text {a }}$ | $\wedge$ | B |
| A | N | N . . . A | A | N |
| A | N | N . . . A | A | N |
| \$ | $\wedge$ | 人 . . . \$ | \$ | $\wedge$ |
| A | \$ | \$ . . . A |  | \$ |

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:

| start from the BTW | sort | we know 1st +last column | rotate (BTW front) | sort |
| :---: | :---: | :---: | :---: | :---: |
| B | A | A . . . B | B A | A N |
| N | A | A . . . N | N A | A N |
| N | A | A . . . N | N A | A \$ |
| $\wedge$ | B | B ... ${ }^{\wedge}$ | $\wedge$ B | B A |
| A | N | N . . . A | A N | N A |
| A | N | N . . . A | A N | N A |
| \$ | $\wedge$ | . \$ | \$ ${ }^{\text {人 }}$ | $\wedge$ B |
| A | \$ | \$ ... A | A \$ | \$ ${ }^{\text {A }}$ |

## Burrows-Wheeler-Transform (BWT) - decoding

| From the BTW the original text can easily decoded: |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| start from the BTW | sort | we know 1st +last column | rotate (BTW front) | sort | rotate (BTW front) |
| B | A | A . . . B | B A | A N | B A N |
| N | A | A . . N | N A | A N | N A N |
| N | A | A . . . N | N A | A \$ | N A \$ |
| $\wedge$ | B | B . . . ${ }^{\text {a }}$ | $\wedge$ B | B A | ${ }^{\wedge} \mathrm{B} A$ |
| A | N | N . . . A | A N | N $A$ | A $\mathrm{N} A$ |
| A | N | N . . . A | A N | N A | A $\mathrm{N} A$ |
| \$ | ^ | ^ . . . \$ | $\$^{\wedge}$ | $\wedge$ - | $\$^{\wedge} \mathrm{B}$ |
| A | \$ | \$ ... A | A \$ | \$ ${ }^{\text {A }}$ | A ${ }^{\text {^ }}$ |

## Burrows－Wheeler－Transform（BWT）－decoding

From the BTW the original text can easily decoded：

| start from the BTW | sort | we know 1st ＋last column | rotate （BTW front） | sort | rotate （BTW front） | sort |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | A | A ．．B | B $\mathbf{A}$ | A N | B A N | A N A |
| N | A | A ．．N | N A | A N | $\mathbf{N} \mathbf{A} \mathbf{N}$ | A NA |
| N | A | A ．．N | N A | A \＄ | N A \＄ | A \＄${ }^{\text {a }}$ |
| ヘ | B | B ．．．${ }^{\wedge}$ | $\wedge$ B | B A | $\wedge B A$ | B A N |
| A | N | N ．．A | A N | N A | A NA | NA N |
| A | N | N ．．A | A N | N A | A $\mathrm{N} A$ | NA \＄ |
| \＄ | $\wedge$ | 人 ．．． $\mathbf{\$}^{\text {d }}$ | \＄${ }^{\text {a }}$ | $\wedge$ B | \＄${ }^{\text {A }} \mathrm{B}$ | $\wedge B A$ |
| A | \＄ | \＄．．．A | A \＄ | \＄${ }^{\text {－}}$ | A \＄ヘ | \＄$\wedge^{\text {A }}$ |

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:

| start from the BTW | sort | we know 1st +last column | rotate (BTW front) | sort | rotate (BTW front) | sort |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | A | A . . . B | B A | A N | B A N | A N A |
| N | A | A . . . N | N A | A N | N A N | ANA |
| N | A | A . . . N | N A | A \$ | N A \$ | A \$ ${ }^{\text {a }}$ |
| $\wedge$ | B | B ... ${ }^{\wedge}$ | $\wedge \mathrm{B}$ | B A | $\wedge$ B A | B A N |
| A | N | N ... A | A N | N A | A $\mathrm{N} A$ | NAN |
| A | N | N . . A | A N | NA | A NA | NA \$ |
| \$ | , | 人 . . ${ }^{\text {S }}$ | \$ ${ }^{\text {® }}$ | $\wedge$ B | $\boldsymbol{\$}^{\wedge} \mathrm{B}$ | $\wedge$ B A |
| A | \$ | \$ . . . A | A ${ }^{\text {S }}$ | \$ ${ }^{\text {A }}$ | A \$ ^ | \$ ^ B |

$$
\begin{aligned}
& \text { add BTW } \\
& \text { and sort } \\
& \text { AN A N } \\
& \text { ANA \$ } \\
& \text { A \$ A B } \\
& \text { BANA } \\
& \text { NANA } \\
& \text { NA \$ A } \\
& \text { ABAN } \\
& \$ \text { ^BA }
\end{aligned}
$$

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:

| start from the BTW | sort | we know 1st +last column | rotate (BTW front) | sort | rotate (BTW front) | sort |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | A | A . . . B | B A | A N | B A N | A N A |
| N | A | A . . . N | N A | A N | N A N | A NA |
| N | A | A . . . N | N A | A \$ | N A \$ | A \$ ${ }^{\text {a }}$ |
| $\wedge$ | B | B ... ${ }^{\wedge}$ | $\wedge \mathrm{B}$ | B A | $\wedge$ B A | B A N |
| A | N | N ... A | A N | N A | A $\mathrm{N} A$ | N A N |
| A | N | N . . A | A N | NA | A NA | NA \$ |
| \$ |  | ^ . . . \$ | \$ ${ }^{\text {® }}$ | $\wedge$ B | $\mathbf{\$ ~}^{\wedge} \mathrm{B}$ | $\wedge$ B A |
| A | \$ | \$ . . . A | A ${ }^{\text {S }}$ | \$ ${ }^{\text {A }}$ | A \$ | \$ ^ B |


| add BTW and sort | add BTW and sort |
| :---: | :---: |
| ANAN | A NANA |
| ANA \$ | ANA\$ ${ }^{\text {A }}$ |
| A $\$^{\wedge} \mathrm{B}$ | A \$ ${ }^{\text {A }} \mathrm{A}$ |
| BANA | BANAN |
| NANA | NANA \$ |
| N A \$ | NA \$ ^ B |
| $\wedge^{\text {B A N }}$ | $\wedge$ BANA |
| $\$^{\wedge} \mathrm{BA}$ | \$ ${ }^{\text {A }}$ B N |

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:

| start from the BTW | sort | we know 1st +last column | rotate (BTW front) | sort | rotate (BTW front) | sort |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | A | A . . . B | B A | A N | B $\mathrm{A} N$ | A N A |
| N | A | A . . . N | N A | A N | NAN | ANA |
| N | A | A . . . N | N $\mathbf{A}$ | A \$ | N A \$ | A \$ ${ }^{\text {a }}$ |
| $\wedge$ | B | B ... ${ }^{\wedge}$ | A B | B A | $\wedge \mathrm{BA}$ | BAN |
| A | N | N . . . A | A N | N $A$ | A $\mathrm{N} A$ | NAN |
| A | N | N . . . A | A N | N $A$ | A NA | N A \$ |
| \$ | - | ^... \$ | $\$^{\wedge}$ | $\wedge$ B | $\wedge^{\wedge} \wedge^{\text {B }}$ | $\wedge$ B A |
| A | \$ | \$ . . A | A \$ | \$ ${ }^{\text {A }}$ | A \$ ${ }^{\text {a }}$ | \$ ^ B |


| add BTW and sort | add BTW and sort | add BTW and sort |
| :---: | :---: | :---: |
| A NAN | ANANA | A NANA \$ |
| ANA \$ | ANA\$ ${ }^{\text {N }}$ | ANA\$ ${ }^{\text {A }}$ B |
| A \$ ${ }^{\wedge} \mathrm{B}$ | A \$ ^ ${ }^{\text {B A }}$ | A \$ ${ }^{\wedge} \mathrm{BAN}$ |
| BANA | BANAN | BANANA |
| NANA | NANA \$ | NANA \$ |
| N A \$ | NA \$ ^ ${ }^{\text {B }}$ | NA \$ ${ }^{\text {A }}$ B A |
| $\wedge \mathrm{BAN}$ | ${ }^{\wedge} \mathrm{BANA}$ | $\wedge$ BANAN |
| \$ ^ B A | \$ ^ BAN | \$ ^ BAN |

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:

| start from the BTW | sort | we know 1st +last column | rotate (BTW front) | sort | rotate (BTW front) | sort |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | A | A . . . B | B $\mathbf{A}$ | A N | B A N | A N A |
| N | A | A . . . N | N $\mathbf{A}$ | A N | N A N | A NA |
| N | A | A . . . N | N $\mathbf{A}$ | A \$ | N A \$ | A \$ ${ }^{\text {a }}$ |
| $\wedge$ | B | B ... ${ }^{\wedge}$ | $\wedge$ A | B A | $\wedge B \mathrm{~A}$ | B A N |
| A | N | N . . . A | A N | N $A$ | A NA | N A N |
| A | N | N . . . A | A N | N A | A $\mathrm{N} A$ | N A \$ |
| \$ | $\wedge$ | ^ . . . \$ | \$ ${ }^{\wedge}$ | $\wedge$ B | $\boldsymbol{\$}^{\wedge} \mathrm{B}$ | $\wedge$ ^ ${ }^{\text {A }}$ |
| A | \$ | \$ . . . A | A ${ }^{\text {S }}$ | \$ ^ | A \$ ^ | \$ ^ B |


| add BTW and sort | add BTW and sort | add BTW and sort | add BTW and sort |
| :---: | :---: | :---: | :---: |
| A NAN | ANANA | ANANA \$ | ANANA \$ |
| A N A \$ | ANA\$ ${ }^{\text {a }}$ | ANA \$ ${ }^{\wedge} \mathrm{B}$ | ANA\$ ${ }^{\text {A }}$ B $A$ |
| A \$ ${ }^{\wedge} \mathrm{B}$ | A \$ ^ B A | A \$ ${ }^{\text {A }} \mathrm{BAN}$ | A \$ ${ }^{\text {A }}$ ANA |
| B A N A | BANAN | BANANA | BANANA \$ |
| NANA | NANA \$ | NANA \$ ${ }^{\text {N }}$ | NANA \$ ${ }^{\text {A }}$ B |
| N A \$ | NA \$ ${ }^{\wedge} \mathrm{B}$ | NA \$ ^ B A | NA \$ ${ }^{\text {A B AN }}$ |
| ^ BAN | ^BANA | ^BANAN | ^BANANA |
| \$ ^ B A | \$ ^ B A N | \$ ^ B A N A | \$ ${ }^{\text {® }}$ B ${ }^{\text {N A N }}$ |

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:

| start from the BTW | sort | we know 1st +last column | rotate (BTW front) | sort | rotate (BTW front) | sort |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | A | A . . . B | B A | A N | B A N | A N A |
| N | A | A . . N | N A | A N | N $\mathrm{A}^{\text {N }}$ | A NA |
| N | A | A . . N | N A | A \$ | N $\mathbf{A}$ \$ | A \$ ${ }^{\text {a }}$ |
| ヘ | B | B ... ${ }^{\wedge}$ | $\wedge$ B | B A | $\wedge$ B A | B A N |
| A | N | N . . A | A N | N $A$ | A N A | NAN |
| A | N | N . . A | A N | N A | A N A | N A \$ |
| \$ | $\wedge$ | ^ . . . \$ | \$ ${ }^{\wedge}$ | $\wedge$ B | $\$^{\wedge} \mathrm{B}$ | $\wedge \mathrm{BA}$ |
| A | \$ | \$ . . . A | A \$ | \$ ${ }^{\text {® }}$ | A ${ }^{\text {^ }}$ | \$ ^ B |


| add BTW and sort | add BTW and sort | add BTW and sort | add BTW and sort | ...until the matrix has its width again |
| :---: | :---: | :---: | :---: | :---: |
| ANAN | ANANA | ANANA \$ | ANANA\$ | ANANA \$ ${ }^{\text {A }} \mathrm{B}$ |
| ANA\$ | ANA\$^ | ANA\$ ${ }^{\wedge} \mathrm{B}$ | ANA\$ ${ }^{\text {A }} \mathrm{BA}$ | ANA\$ ${ }^{\text {A }}$ BAN |
| A \$ ${ }^{\wedge} \mathrm{B}$ | A \$ ^ ${ }^{\text {B A }}$ | A \$ ${ }^{\wedge} \mathrm{BAN}$ | A \$ ^ BANA | A \$ ${ }^{\text {c }}$ ANAN |
| BANA | BANAN | BANANA | BANANA \$ | BANANA\$^ |
| NANA | NANA \$ | NANA\$ | NANA\$ ${ }^{\text {A }}$ B | NANA\$ ${ }^{\text {A B A }}$ |
| N A \$ | NA \$ ${ }^{\wedge} \mathrm{B}$ | NA \$ ^ B A | NA \$ ^ B A N | NA\$ ${ }^{\text {A B ANA }}$ |
| 人 BAN | $\wedge$ BANA | $\wedge$ BANAN | ${ }^{\wedge} \mathrm{B} A N A N A$ | $\wedge$ BANANA \$ |
| \$ ^ B A | \$ ^ BAN | \$ ^ BANA | \$ ^ BANAN | \$ ${ }^{\text {A B A ANA }}$ |

## Burrows-Wheeler-Transform (BWT) - decoding

From the BTW the original text can easily decoded:

| start from the BTW | sort | we know 1st +last column | rotate (BTW front) | sort | rotate (BTW front) | sort |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B | A | A . . B | B A | A N | B A N | A N A |
| N | A | A . . N | N $\mathbf{A}$ | A N | N $\mathbf{A} \mathbf{N}$ | A N A |
| N | A | A . . N | N A | A \$ | N A \$ | A \$ ${ }^{\text {a }}$ |
| $\wedge$ | B | B ... ${ }^{\text {a }}$ | $\wedge$ B | B A | $\wedge$ B A | B A N |
| A | N | N . . . A | A N | N A | A NA | NA N |
| A | N | N . . A | A N | N A | A NA | N A \$ |
| \$ | $\wedge$ | 人 . . \$ | \$ ${ }^{\text {a }}$ | $\wedge$ B | \$ ${ }^{\text {A }}$ | $\wedge$ B A |
| A | \$ | \$ ... A | A \$ | \$ ${ }^{\text {® }}$ | A \$ ${ }^{\text {a }}$ | \$ ${ }^{\text {A }} \mathrm{B}$ |


| add BTW and sort | add BTW and sort | add BTW and sort | add BTW and sort | ...until the matrix has its width again |
| :---: | :---: | :---: | :---: | :---: |
| A N A N | ANANA | ANANA \$ | ANANA \$ | ANANA \$ ${ }^{\text {A }} \mathrm{B}$ |
| A N A \$ | ANA\$ ${ }^{\text {a }}$ | ANA\$ ${ }^{\text {N }} \mathrm{B}$ | ANA\$ ${ }^{\text {N }} \mathrm{BA}$ | ANA\$ ${ }^{\text {A }}$ AN |
| A \$ ${ }^{\wedge} \mathrm{B}$ | A \$ ^ BA | A \$ ${ }^{\wedge} \mathrm{BAN}$ | A \$ ${ }^{\wedge} \mathrm{BANA}$ | A \$ ${ }^{\text {c }}$ BNAN |
| BANA | BANAN | BANANA | BANANA \$ | BANANA\$ |
| N A NA | NANA \$ | NANA\$ ${ }^{\text {N }}$ | NANA\$ ${ }^{\text {A }}$ B | NANA \$ ${ }^{\text {A B A }}$ |
| N A \$ | NA \$ ^ B | NA \$ ^ B A | NA \$ ${ }^{\text {® }} \mathrm{BAN}$ | NA\$ ${ }^{\text {A }}$ BANA |
| $\wedge \mathrm{BAN}$ | $\wedge$ B ANA | ${ }^{\wedge} \mathrm{BANAN}$ | ${ }^{\wedge} \mathrm{BANANA}$ | ${ }^{\wedge} \mathrm{BANANA}$ A |
| \$ ^ B A | \$ ^ B A N | \$ ^ B A N A | \$ ^ B A N A N | \$ ^ B A N A A |

The original text can be found in the line starting with ^ and ending with \$.

## Burrows-Wheeler-Transform (BWT) - some observations

$\wedge^{1} B^{2} A^{3} N^{4} A^{5} N^{6} A^{7} \$^{8}$

## Burrows-Wheeler-Transform (BWT) - some observations

```
    ^ ^1 B ' A ' N N
\mp@subsup{}{}{3}}\mathbf{A}\mathbb{N A N A $ ^ B}\mp@subsup{\mathbf{B}}{}{2
\mp@subsup{}{}{5}\mathbf{A N A $ ^ B A N'N}
7}\mathbf{A}$^|\mathcal{A}N\mathbb{A}\mp@subsup{\mathbf{N}}{}{6
2}\mp@subsup{}{}{2}\textrm{B}A\mathbb{N A NA $ ^1
\mp@subsup{}{}{4}\mathbf{N}A\mathbb{NAS ^ B A}\mp@subsup{}{}{3}
\mp@subsup{}{}{6}\mathbf{N}A$ ^ B A N A
1^ BANANA $8
8
```


## Burrows-Wheeler-Transform (BWT) - some observations



- a letter in the 1 st column is easy to find, because they are sorted


## Burrows-Wheeler-Transform (BWT) - some observations

$\wedge^{1} B^{2} A^{3} N^{4} A^{5} N^{6} A^{7} \$^{8}$


```
{ } ^ { 5 } \mathbf { A } - 1
7}\mathbf{A
2}\textrm{B}-\textrm{A}-\textrm{N}-\textrm{A
4}\mathbf{N
```



```
1^-B-A-M-K-N-> $ }\mp@subsup{}{}{8
8}\boldsymbol{$}-A-B-2-1\mp@code{Hz-> A
```

- a letter in the 1st column is easy to find, because they are sorted
- the letter in the last column preceeds the one in the 1st column (thus, searching for words starts at the last letter)


## Burrows-Wheeler-Transform (BWT) - some observations



- a letter in the 1st column is easy to find, because they are sorted
- the letter in the last column preceeds the one in the 1st column (thus, searching for words starts at the last letter)
- the order of occurrence of a single letter in the last and the 1 st column is the same (the $2 \mathrm{nd} A$ in the one is the $2 \mathrm{nd} A$ in the other)


## Burrows-Wheeler-Transform (BWT) - some observations

$\wedge^{1} B^{2} A^{3} N^{4} A^{5} N^{6} A^{7} \$^{8}$


- a letter in the 1st column is easy to find, because they are sorted
- the letter in the last column preceeds the one in the 1st column (thus, searching for words starts at the last letter)
- the order of occurrence of a single letter in the last and the 1st column is the same (the $2 \mathrm{nd} A$ in the one is the $2 \mathrm{nd} A$ in the other)


## Burrows-Wheeler-Transform (BWT) - searching

Searching from the last letter to the first of the search string ( $q=A N A$ ):

| query: | $\mathrm{q}=\mathbf{A} \mathbf{N}$ |
| :---: | :---: |
| ${ }^{3} \mathrm{~A}$ | \$ ${ }^{\text {B }}$ |
| ${ }^{5} \mathrm{~A}$ N | $\wedge B A \mathbf{N}^{4}$ |
| ${ }^{7} \mathbf{A}$ \$ | A $N$ A $\mathbf{N}^{6}$ |
| ${ }^{2} \mathrm{~B}$ A | NA\$ ${ }^{1}$ |
| ${ }^{4} \mathrm{~N}$ A | \$ ^ $\mathrm{B} \mathbf{A}^{3}$ |
| ${ }^{6} \mathrm{~N}$ A | $\mathrm{BAN} \mathbf{A}^{5}$ |
| 1^ B | $\mathrm{A} N \mathrm{~A} \mathbf{\$}^{8}$ |
| ${ }^{8}$ \$ ${ }^{\text {A }}$ | NAN $\mathbf{A}^{7}$ |

## Burrows-Wheeler-Transform (BWT) - searching

Searching from the last letter to the first of the search string ( $q=A N A$ ):

| query: $\quad \mathrm{q}=\mathbf{A N} \mathbf{A}$ | find last $\quad \mathrm{q}=\mathbf{A N}$ |
| :---: | :---: |
| ${ }^{3} \mathbf{A N A N A}$ ( ${ }^{\text {® }} \mathbf{B}^{2}$ | ${ }^{3} \mathrm{~A} N \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{~B}^{2}$ |
| ${ }^{5} \mathbf{A N A}$ ¢ ${ }^{\text {A }} \mathrm{BA} \mathbf{N}^{4}$ | ${ }^{5} \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{BA} \mathbf{N}^{4}$ |
| ${ }^{7} \mathbf{A} \$ \wedge \operatorname{BANA} \mathbf{N}^{6}$ | ${ }^{7} \mathrm{~A} \$ \wedge \mathrm{~B} \boldsymbol{A} N \mathbb{A} \mathbf{N}^{6}$ |
| ${ }^{2} \mathrm{BANANA}$ ( ${ }^{1}$ | ${ }^{2} \mathrm{~B} A \mathbb{N} \mathbb{A} \mathbb{A}$ \$ |
| ${ }^{4} \mathbf{N} \boldsymbol{A} N \mathrm{~A} \$ \wedge \mathrm{~B} \mathbf{A}^{3}$ | ${ }^{4} \mathbf{N} \boldsymbol{A} N \mathrm{~A} \$ \wedge \mathrm{~B} \mathbf{A}^{3}$ |
| ${ }^{6} \mathrm{NA}$ ¢ ${ }^{\text {A }} \mathrm{BAN} \mathrm{A}^{5}$ | ${ }^{6} \mathbf{N}$ A \$ ${ }^{\wedge} \mathrm{BANA} \mathbf{A}^{5}$ |
| ${ }^{10} \mathrm{BANANA} \mathbf{\$}^{8}$ | ${ }^{1}$ ^ $B A N A N A \mathbf{S}^{8}$ |
| ${ }^{\mathbf{\$}}{ }^{\wedge} \mathrm{BANAN} \mathbf{A}^{7}$ | ${ }^{8} \mathbf{\$}$ ^ BANAN |

## Burrows-Wheeler-Transform (BWT) - searching

Searching from the last letter to the first of the search string ( $q=A N A$ ):

| $\mathrm{q}=\mathbf{A N A}$ | $\mathrm{q}=\mathrm{A}$ | $\mathrm{q}=$ |
| :---: | :---: | :---: |
| NA B | A ${ }^{\text {N }}$ A |  |
| ${ }^{5} \mathbf{A} N \mathrm{~A} \$ \wedge \mathrm{BA} \mathbf{N}^{4}$ | ${ }^{5} \mathbf{A} N \mathrm{~A}$ \$ ${ }^{\text {a }} \mathrm{BA} \mathbf{N}^{4}$ | ${ }^{5} \mathbf{A}-1-2-\$$ - |
| ${ }^{7} \mathbf{A} \$ \wedge \operatorname{BANA} \mathbf{N}^{6}$ | ${ }^{7} \mathrm{~A} \$ \wedge \mathrm{BANA} \mathbf{N}^{6}$ | ${ }^{7} \mathbf{A}-{ }^{\text {a }}$ - |
| ${ }^{2} \mathrm{BANANA}$ ( ${ }^{1}$ | ${ }^{2} \mathrm{~B} A \mathbb{N} \mathbb{A} \mathrm{~N} A$ | ${ }^{2} \mathrm{~B}$ |
| ${ }^{4} \mathbf{N A N A}$ ¢ ${ }^{\text {A B }} \mathbf{A}^{3}$ | ${ }^{4} \mathbf{N A N A}$ ¢ ${ }^{\text {A B }} \mathbf{A}^{3}$ | ${ }^{4} \mathbf{N A}{ }^{\text {A }}$ - $\mathcal{A} \$ \wedge B \mathbf{A}^{3}$ |
| ${ }^{6} \mathbf{N} A \$ \wedge B A N \mathbf{A}^{5}$ | ${ }^{6} \mathbf{N} A \$ \wedge B A N \mathbf{A}^{5}$ | ${ }^{6} \mathrm{~N}{ }^{\prime}$ \$ ${ }^{\text {¢ }} \mathrm{BANA} \mathrm{A}^{5}$ |
| BANANA ${ }^{8}$ | $1 \wedge$ BANANA ${ }^{\text {® }}$ | $1 \wedge$ BANANA ${ }^{8}$ |
| $\mathbf{\$}^{\wedge} \mathrm{BANANA} \mathbf{A}^{7}$ | \$ | ${ }^{8}$ \$ ${ }^{\text {B A NAN }}$ |

## Burrows-Wheeler-Transform (BWT) - searching

Searching from the last letter to the first of the search string ( $q=A N A$ ):

| query: $\quad q=A \sim A$ |  | $\begin{aligned} & \text { to next } \\ & \text { position }\end{aligned} \quad q=A N A$ |
| :---: | :---: | :---: |
| ${ }^{3} \mathbf{A} N \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{~B}^{2}$ | ${ }^{3} \mathrm{~A} N \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{~B}^{2}$ | ${ }^{3} \mathbf{A}-\mathrm{H}-\mathrm{A}-\mathrm{N}-\mathrm{A}-\$-\wedge-\mathbf{B}^{2}$ |
| ${ }^{5} \mathbf{A} N \mathrm{~A} \$ \wedge \mathrm{BA} \mathbf{N}^{4}$ | ${ }^{5} \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{BA} \mathbf{N}^{4}$ | ${ }^{5} \mathbf{A}-\mathrm{H}-\mathrm{A}-\$-\wedge-B<-\mathbf{N}^{4}$ |
| ${ }^{7} \mathbf{A} \$ \wedge$ B A N A N ${ }^{6}$ | ${ }^{7} \mathbf{A} \$ \wedge$ B A NA $\mathbf{N}^{6}$ |  |
| ${ }^{2} \mathbf{B A N A N A} \mathrm{BNA}^{1}$ | ${ }^{2} \mathrm{BANANA} \$ \mathrm{Al}^{1}$ | ${ }^{2} \mathrm{BA} \mathrm{N}^{\prime} \mathrm{A}^{\prime}$ NA'\$ ${ }^{1}$ |
| ${ }^{4} \mathbf{N A N A}$ \$ ${ }^{\text {A }} \mathrm{BA}^{3}$ | ${ }^{4} \mathbf{N A N A} \$ \wedge B \mathbf{A}^{3}$ | ${ }^{4} \mathbf{N A} \mathrm{~N}^{\prime} \mathbf{A} \$ \wedge B \mathbf{A}^{3}$ |
| ${ }^{6} \mathbf{N} \boldsymbol{A}$ \$ $\wedge$ BAN $\mathbf{A}^{5}$ | ${ }^{6} \mathbf{N} \mathbf{A} \$ \wedge$ B A N $\mathbf{A}^{5}$ | ${ }^{6} \mathrm{NA}^{\prime}$ \$ ${ }^{\text {a }} \mathrm{BANA} \mathrm{A}^{5}$ |
| ${ }^{1} \wedge \mathrm{BANANA} \mathbf{\$}^{8}$ | ${ }^{1 \times \mathrm{BANANA} \mathbf{\$}^{8}}$ | ${ }^{1 \times \mathrm{B}} \mathrm{ANANA} \mathrm{\mathbf{S}}^{8}$ |
| ${ }^{8} \boldsymbol{\$} \wedge \mathrm{BANAN} \mathbf{A}^{7}$ | ${ }^{8} \boldsymbol{\$} \wedge \mathrm{BA} N \mathrm{~A} N \mathbf{A}^{7}$ | ${ }^{8} \mathbf{\$} \wedge$ B $A N A N \mathbf{A}^{7}$ |
| check 2nd $q=A N A$ last letter |  |  |
| ${ }^{3} \mathbf{A} N \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{~B}^{2}$ |  |  |
| ${ }^{5} \mathbf{A} N \mathrm{~A}$ \$ $\wedge$ BA $\mathbf{N}^{4}$ |  |  |
| ${ }^{7} \mathbf{A} \$ \wedge$ B A N $\mathrm{N}^{6}$ |  |  |
| ${ }^{2} \mathbf{B A N A N A} \$ \mathrm{Nr}^{1}$ |  |  |
| ${ }^{4} \mathbf{N} \mathbf{A} N \mathrm{~A} \$ \wedge \mathrm{~B} \mathbf{A}^{3}$ |  |  |
| ${ }^{6} \mathrm{NA}$ \$ ${ }^{\text {A }} \mathrm{BAN} \mathbf{A}^{5}$ |  |  |
| ${ }^{1} \wedge \mathrm{BANANA} \mathbf{\$}^{8}$ |  |  |
| ${ }^{8} \boldsymbol{\$} \wedge \mathrm{BANA}$ ( $\mathbf{A}^{7}$ |  |  |

## Burrows-Wheeler-Transform (BWT) - searching

Searching from the last letter to the first of the search string ( $q=A N A$ ):

| query: $\quad \mathbf{q}=\mathbf{A} \mathbf{N} \mathbf{A}$ | $\underset{\text { letter last }}{\text { find la }} \quad \mathrm{q}=\mathbf{A N A}$ | $\begin{aligned} & \text { to next } \\ & \text { position }\end{aligned} \quad q=A N A$ |
| :---: | :---: | :---: |
| ${ }^{3} \mathbf{A} N \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{~B}^{2}$ | ${ }^{3} \mathrm{~A} N \mathrm{ANA}$ \$ ${ }^{\text {( }}{ }^{2}$ | ${ }^{3} \mathbf{A}-\mathrm{N}-\mathrm{A}-\mathrm{N}-\mathrm{A}-\$+-\mathbf{B}^{2}$ |
| ${ }^{5} \mathbf{A} N \mathrm{~A} \$ \wedge \mathrm{BA} \mathbf{N}^{4}$ | ${ }^{5} \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{BA} \mathbf{N}^{4}$ | ${ }^{5} \mathbf{A}-\mathrm{H}-\mathrm{A}-\mathrm{S}^{+}$ |
| ${ }^{7} \mathbf{A} \$ \wedge$ B A NA $\mathbf{N}^{6}$ | ${ }^{7} \mathbf{A} \$ \wedge \mathrm{BANA} \mathbf{N}^{6}$ | ${ }^{7} \mathbf{A}-\$-\stackrel{\text { B }}{ }$ |
| ${ }^{2}$ B A NANA \$ ${ }^{1}$ | ${ }^{2} \mathrm{BANANA} \$ \mathbf{N 1}^{1}$ | ${ }^{2} \mathrm{BAN} \mathrm{N}^{\prime} \mathrm{NHA}^{\prime}$ \$ 1 |
| ${ }^{4} \mathbf{N A N A}$ \$ ${ }^{\text {A }} \mathrm{B} \mathbf{A}^{3}$ | ${ }^{4} \mathbf{N A N A} \$ \wedge B \mathbf{A}^{3}$ |  |
| ${ }^{6} \mathbf{N} \boldsymbol{A}$ \$ $\wedge$ B A $\mathrm{A}^{5}$ | ${ }^{6} \mathbf{N} \mathbf{A} \$ \wedge B A N \mathbf{A}^{5}$ |  |
| ${ }^{1} \boldsymbol{\wedge} \mathrm{BANANA} \mathbf{\$}^{8}$ | ${ }^{1 \times} \mathrm{BANANA} \mathbf{\$}^{8}$ | ${ }^{1} \boldsymbol{A} \mathrm{~B}$ ( $\mathrm{A} N \mathrm{~A} \mathbf{\$}^{8}$ |
| ${ }^{8} \boldsymbol{\$} \wedge B A N A N \mathbf{A}^{7}$ | ${ }^{8} \overline{\boldsymbol{\$} \wedge} \mathrm{BANA}$, $\mathrm{A}^{7}$ | ${ }^{8} \overline{\boldsymbol{\$}} \wedge \mathrm{BANA} \mathrm{A}^{7}$ |
| check 2nd $q=A N A$ last letter | $\begin{aligned} & \text { to next } \\ & \text { position } \end{aligned} \quad q=\text { A N A }$ |  |
| ${ }^{3} \mathbf{A} N \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{~B}^{2}$ | ${ }^{3} \mathbf{A} N \mathrm{~A} N \mathrm{~A} \$ \wedge \mathrm{~B}^{2}$ |  |
| ${ }^{5} \mathbf{A} N \mathrm{~A}$ \$ ${ }^{\text {a }}$ BA $\mathbf{N}^{4}$ | ${ }^{5} \mathbf{A} \mathrm{R}^{\prime} \mathbf{A}$ \$ $\mathrm{T}^{\text {B A }} \mathbf{N}^{4}$ |  |
| ${ }^{7} \mathbf{A} \$ \wedge$ B A NA $\mathbf{N}^{6}$ | ${ }^{7} \mathbf{A} \$ \cdots$ 它ANA $\mathbf{N}^{6}$ |  |
| ${ }^{2} \mathbf{B A N A N A}$ \$ $\mathbf{N 1}^{1}$ |  |  |
| ${ }^{4} \mathbf{N} \mathbf{A} N \mathrm{~A} \$ \wedge \mathrm{~B} \mathbf{A}^{3}$ | ${ }^{4} \mathbf{N}-\mathbf{A}-\mathbb{N}-\mathbf{A}-\$ \cdots, ~-\mathbf{A}^{3}$ |  |
| ${ }^{6} \mathbf{N} \mathbf{A} \$ \wedge$ B $\mathrm{A} N \mathbf{A}^{5}$ | ${ }^{6} \mathbf{N}-2-\$-A-B-2, N$ A ${ }^{5}$ |  |
| 1^BANANA ${ }^{8}$ | ${ }^{1 \times} \mathrm{BANANA} \mathbf{\$}^{8}$ |  |
| ${ }^{8} \mathbf{\$} \wedge$ BANAN $\mathbf{A}^{7}$ | ${ }^{8} \mathbf{\$} \wedge \mathrm{BA} N \mathrm{AN} \mathbf{A}^{7}$ |  |

## Burrows-Wheeler-Transform (BWT) - searching

Searching from the last letter to the first of the search string ( $q=A N A$ ):

| $\mathrm{q}=\mathrm{A} \times \mathrm{A}$ | $\mathrm{q}=\mathrm{A} \times \mathrm{A}$ | t |
| :---: | :---: | :---: |
| $\boldsymbol{A} \mathbb{A} \${ }^{\wedge} \mathrm{B}^{2}$ | ${ }^{3} \mathbf{A} N \mathrm{ANA}$ ( ${ }^{\text {® }} \mathrm{B}^{2}$ |  |
| ${ }^{5} \mathbf{A N A}$ ¢ ${ }^{\text {A }} \mathrm{BA} \mathbf{N}^{4}$ | ${ }^{5} \mathbf{A} N \mathrm{~A} \$ \wedge \mathrm{BA} \mathbf{N}^{4}$ | $\mathrm{N}^{4}$ |
| ${ }^{7} \mathbf{A} \$ \wedge \operatorname{BANA} \mathbf{N}^{6}$ | ${ }^{7} \mathbf{A} \$ \wedge \mathrm{BANA} \mathbf{N}^{6}$ | ${ }^{7} \mathbf{A}-{ }^{\text {d }}$ |
| ${ }^{2} \mathrm{BANANA}$ ( ${ }^{1}$ | ${ }^{2}$ B ANANA \$ ${ }^{1}$ |  |
| ${ }^{4} \mathrm{~N}$ | ${ }^{4} \mathbf{N A N A}$ ¢ ${ }^{\text {A }} \mathrm{B} \mathbf{A}^{3}$ | ${ }^{4} \mathbf{N A} \mathbf{N}^{\prime} \mathbf{A} \$ \wedge B \mathbf{A}^{3}$ |
| ${ }^{6} \mathbf{N A}$ ¢ ${ }^{\text {A }} \mathrm{BAN} \mathbf{A}^{5}$ | ${ }^{6} \mathbf{N}$ A \$ ${ }^{\text {A }} \mathrm{BANA} \mathbf{A}^{5}$ | ${ }^{6} \mathrm{NA}^{\prime}$ \$ ${ }^{\text {¢ }} \mathrm{BANA} \mathrm{A}^{5}$ |
| ${ }^{1} \wedge$ B A NAN | ${ }^{1}$ ^ $B A N A N A \mathbf{S}^{8}$ | ${ }^{1} \wedge \mathrm{BANANA} \mathbf{\$}^{8}$ |
| ${ }^{8}$ \$ ${ }^{\text {B }} \mathrm{BNAN} \mathbf{A}^{7}$ | ${ }^{8}$ \$ ${ }^{\text {A B A N A N } \mathbf{A}^{7}}$ | ${ }^{8}$ \$ ${ }^{\text {A }} \mathrm{BANAN} \mathrm{A}^{7}$ |
| $\begin{aligned} & \text { check 2nd } \\ & \text { last letter }\end{aligned} \quad q=A N A$ | $\begin{aligned} & \text { to next } \\ & \text { position }\end{aligned} \quad q=A N A$ | $\begin{aligned} & \text { found } \\ & \text { twice! }\end{aligned} \quad q=A$ N A |
| ${ }^{3} \mathbf{A} \sim \mathbb{A} \sim \mathrm{~A}$ \$ |  | ${ }^{3} \mathrm{~A} N \mathrm{ANA}$ \$ ${ }^{\text {® }}{ }^{2}$ |
| ${ }^{5} \mathbf{A} N \mathbb{A} \$ \wedge B A \mathbf{N}^{4}$ | ${ }^{5} \mathbf{A} \mathrm{~N}^{\prime} \boldsymbol{A}, \$ \wedge \mathrm{BA} \mathbf{N}^{4}$ | ${ }^{5} \mathrm{~A} N \mathrm{~A}$ \$ ${ }^{\text {¢ }} \mathrm{B} \boldsymbol{A} \mathbf{N}^{4}$ |
| ${ }^{7} \mathbf{A} \$ \wedge$ ¢ ${ }^{\text {A N A N }}{ }^{6}$ |  | ${ }^{7} \mathbf{A} \$ \wedge \mathrm{BANA} \mathbf{N}^{6}$ |
| ${ }^{2} \mathrm{BANANA}$ ( ${ }^{1}$ | ${ }^{2} \mathrm{BANANNA}$ | ${ }^{2} \mathrm{BANANA} \mathrm{Na}^{1}$ |
| ${ }^{4} \mathbf{N A N A}$ ( ${ }^{\text {A B }} \mathbf{A}^{3}$ |  | ${ }^{4} \mathbf{N A N A}$ A ${ }^{\text {A }} \mathbf{A}^{3}$ |
| ${ }^{6} \mathrm{NA} \$ \wedge$ ¢ $\mathrm{AN} \mathbf{A}^{5}$ | ${ }^{6} \mathbf{N}-2-\$-A-B-2 A+\mathbf{A}^{5}$ | ${ }^{6} \mathbf{N A}$ \$ ${ }^{\text {A }} \mathrm{BANA} \mathrm{A}^{5}$ |
| $B \mathbb{N} \mathbb{A} \mathbb{A} \mathbf{\$}^{8}$ | ${ }^{1 \wedge} \mathrm{BANANA} \mathbf{\$}^{8}$ |  |
| ${ }^{\wedge} \mathrm{BANA} N \mathbf{A}^{7}$ | ${ }^{\text {¢ }}$ ^ BANA NA | ${ }^{8}$ \$ BANANX |

## Burrows-Wheeler-Transform (BWT) - approximate matches

- this way exact matches can be found easily
- to find approximate matches,
- everytime a mismatch is detected,
- a backtrace is done, introducing changes at any position
- at the beginning only one change and later more if still no match is found


## BWT-based mapping



- BWT-based is harder to implement than seed based approaches
- however, it is less memory intense (only (about 1-2GB for the human genome of 3 Gbp) and much faster
- on the other hand, seed based approaches have been shown to be much more sensitive, and thus able to match more reads correctly

